Mathematics
Class XII

Binary Operations

1. Show that the operation * defined on $R-\{0\}$ by $a*b=|ab|$ is a binary operation. Show also that * is commutative and associative.

Ans.

We have the operation $a*b=|ab|$ on $R-\{0\}$

Let, $a,b \in R-\{0\}$ any two element.

$\therefore$ $a,b \in R-\{0\}$ [$\because$ Multipation is a binary operation on R]

$\Rightarrow |ab| \in R-\{0\}$

$\therefore$ $a*b \in R-\{0\}$; for all $a,b \in R-\{0\}$.

$\therefore$ R – {0} is closed with respect to *.

So, * is a binary operation on R – {0}.

Commutativity:

Let, $a,b \in R-\{0\}$

$\therefore$ $a*b=|ab|$ and $b*a=|ba|=|ab|$

$\therefore$ $a*b=b*a$ সকল $a*b \in R-\{0\}$ এর জন্য।

$\therefore$ * is commutative binary operation on R – {0}

Associativity:

Let, $a,b,c \in R-\{0\}$

$\therefore$ $(a*b)*c=|ab|*c=||ab|.c|=|abc|$

And $a*(b*c)=a*|bc|=|a.|bc||=|abc|$

$\therefore$ $(a*b)*c=a*(b*c)$ for all $a,b,c \in R-\{0\}$.

$\therefore$ * is associative binary operation on R – {0}.

2. '*' is a binary operation defined on Z set of all integers as $a*b=a+b+1$ for all $a,b \in Z$.

(i) Find the identity element of '*' (ii) Find the inverse of an element $a \in Z$.

Ans.

Existence of identity element:

Let e be an identity element in Z with respect to '*'

So, $a*e=e*a=a$ for all $a \in Z$.

$\Rightarrow a+e+1=e+a+1=a$ for all $a \in Z$. $\Rightarrow e=-1 \in Z$.

So the identity element in Z with respect to '*' on Z is –1.

Extence if inverse:

Let, $a \in Z$ and let its inverse be b.

$\therefore \quad a * b = e \Rightarrow a + b + 1 = -1 \Rightarrow b = -(2+a) \quad [\because e = -1]$

$\because \quad 2 \in Z, a \in Z \;\therefore (2+a) \in Z \Rightarrow -(2+a) \in Z$

So each $a \in Z$ has an inverse given by $-(2+a)$.

3. Let A = {1,–1, i, – i}, where $i = \sqrt{-1}$. Prepare the composition table corresponding to the binary operation usul 'multiplication' on A and discuss its important properties.

Ans.

Solution:

We have

$1 \times 1 = 1$	$1 \times (-1) = -1$	$1 \times i = i$	$1 \times (-i) = -i$
$(-1) \times 1 = -1$	$(-1) \times (-1) = 1$	$(-1) \times i = -i$	$(-1) \times (-i) = i$
$i \times 1 = i$	$i \times (-1) = -i$	$i \times i = -1$	$i \times (-i) = 1$
$(-i) \times 1 = -i$	$(-i) \times (-1) = i$	$(-i) \times i = 1$	$(-i) \times (-i) = -1$

$\therefore$ the required composition table w.r.t. usual 'multiplication' on A is given below :

x	1	–1	i	–i
1	1	–1	i	–i
–1	–1	1	–i	i
i	i	–i	–1	1
–i	–i	i	1	–1

Observing the above composition table we note its following important properties :

(i) $\because$ $4^2 = 16$ entries of the table are elements of A. Hence usual 'multiplication' is a binary operation on A.

(ii) The table is symmetric w.r.t. its principal diagonal.

$\therefore$ 'multiplication' is commutative on A.

(iii) First row of the composition table coincides with the top most row while its first column coincides with left most column and these first row and first column of the table intersect at 1. Hence, 1 is the identity element for 'multiplication' on A.

(iv) Since the identity element 1 occurs in each row and each column of the composition table, hence every element of A is invertible.

Clearly, $1^{-1} = 1, (-1)^{-1} = -1, i^{-1} = -i, (-i)^{-1} = i$